International Journal of Thermal Sciences 48 (2009) 2355–2364

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Short time step heat flow calculation of building constructions based on frequency-domain regression method

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ARTICLE INFO

Article history:
Received 10 January 2009
Received in revised form
28 April 2009
Accepted 19 May 2009
Available online 17 June 2009

Keywords:
Short time step
Heat flow calculation
CTF coefficients
FDR method

ABSTRACT

Short time step heat flow calculation of building constructions is often needed for practical applications. Conventional methods such as state-space method and root-finding method may produce unstable conduction transfer function (CTF) coefficients at short time steps, and thus result in unstable heat flow calculation through building constructions. Frequency-domain regression (FDR) method is a newly developed method for computing CTF coefficients efficiently by representing the real building construction system with equivalent polynomial *s*-transfer functions. Previous studies on this method mainly addressed CTF coefficients at the conventional time step of 3600 s and the performance of heat flow calculation using these coefficients. This paper presents the investigation on the performance of CTF coefficients at various short time steps based on FDR method, and the performance of the heat flow calculation using these coefficients. The results show that FDR method can produce stable CTF coefficients at various time steps for most building constructions, and the calculated heat flows using these coefficients are of high accuracy.

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1. Introduction

Conduction transfer function (CTF) method [1] is widely used for heat flow calculation of buildings for system design or system operation and control since CTF coefficients relate the desired outputs at a moment to the previous inputs through a set of coefficients. In building energy simulation packages such as DOE-2 [2], EnergyPlus [3] and BLAST [4] etc., CTF method is the most popular while other methods may also be used [5–7].

The mainly used methods resulting in CTF coefficients are conventional root-finding method, state-space method, and frequency-domain regression method. Conventional root-finding method is to find the poles of the hyperbolic *s*-transfer function of a building construction for calculating response factors and/or CTF coefficients [8]. State-space method [9–11] is to obtain CTF coefficients directly through numerical computation by relating the interior and exterior boundary temperatures to the inside and outside surface heat fluxes at each node of a multilayer slab through the use of matrix algebra. These nodes are artificially placed by enforcing a finite difference grid over the various layers in the building construction of concern. Frequency-domain regression (FDR) method [12–16] is to simplify the complex root-finding process in conventional root-finding method by approximating the hyperbolic *s*-transfer function of a building construction with a polynomial *s*-transfer function based on equivalent frequency response characteristics. Li et al. [17] further investigated the applicability of FDR method by comparing with conventional root-finding method and state-space method when Fourier number and thermal structure factor are varied. The results show that FDR method has more robustness and reliability than the other two methods.

CTF coefficients are often needed to be customized with different time step intervals for practical applications. For the purpose of designing buildings and air-conditioning systems such as DOE-2, CTF coefficients with the time step of 3600 s are sufficient to calculate the heat flux through building constructions or building internal mass. For the purpose of control simulation of the air-conditioning system etc., CTF coefficients with shorter time steps are more preferable to simulate the heat flow flux through

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doi:10.1016/j.ijthermalsci.2009.05.005

Nomenclature

A, B, D	elements of transmission matrix
A_x, B_x	polynomial
A_Y, B_Y	polynomial
A_Z, B_Z	polynomial
a, b, d	transfer function coefficients
CTF	conduction transfer function
Error	relative error, %
G, G'	transfer function
Q, q	heat flow, W m^{-2}
S	Laplace variable or roots
T	temperature, °C
U	transmittance value
Y	cross thermal response factor, W m^{-2} K^{-1}

Greek symbols

Δ	time step, s
σ	residues
η	slope of excitation function

Subscripts

In	inside
Out	outdoor, outside
sol-air	solar air
Total	total
X	associated with external heat conduction
Y	associated with cross heat conduction
Z	associated with internal heat conduction

building constructions resulting in cooling load. EnergyPlus [3] provides six options of time steps for heat balance calculation, i.e., 3600 s, 1800 s, 1200 s, 900 s, 720 s and 600 s. IBLAST [18], a research version of BLAST with HVAC systems integrated into the building load simulation, also uses multiple time step approach for heat balance simulation. In this package, the state-space method developed by Seem [11] is used to improve the stability of CTF coefficients at shorter time steps, but not enough to allow IBLAST to run at the time step of 360 s without restricting the types of surfaces that could be used. In fact, CTF coefficient series derived at short time steps from root-finding method or state-space method may become progressively unstable as the time step decreases. The Ref. [3] pointed out that the number of terms in CTF series grows when the time step for calculating the CTF series gets shorter. These CTF series become unstable, and finally the entire simulation becomes diverging. It is also pointed out that it is difficult to compute stable CTF coefficients for the time step less than 900 s for heavy constructions. Therefore, 900 s is recommended to be used in EnergyPlus package for heat balance calculation through building constructions although more short time steps could be chosen.

Heat flow calculation through building envelope is usually needed to interact with building HVAC system. Building heat transfer is a relatively slow response process when comparing with the heat transfer of most HVAC systems having the characteristics of quick responses. For instance, VAV system and AHU system are often controlled at minute level. Although EnergyPlus [3] only provides six options of time step at the input interface of the simulation package without the time steps of less than 600 s, it presents three methods to compute the heat flow at arbitrary time steps by utilizing the CTF coefficients of a structure calculated at specified time step. These three methods may be called "Multiple, staggered time history scheme", "Sequential interpolation of new histories", and "Master history with interpolation". For control and optimization simulation, many researchers have alternatives to simulate building heat transfer [19–22]. House [20] developed a system approach for optimal control of HVAC system. In this approach, a first-order thermal network model was used to represent the building heat transfer for facilitating the interaction of the building heat transfer and operation of HVAC system through the zonal air. Mossolly et al. [22] also used simplified thermal network models to represent the heat transfer of external walls and internal walls of a multi-zone building for developing optimal control strategy of an air-conditioning system. The simulation step of 1 s was used for the heat transfer calculation of the building structure and operation of the air-conditioning system.

Frequency-domain regression method [12–16] is a recently developed alternative for computing CTF coefficients efficiently and effectively. However, the previous studies on this method only addressed the performance of CTF coefficients at the conventional time step of 3600 s and the performance of heat flow calculation using these coefficients. The main aim of this study is to present the investigation on the performance of CTF coefficient series at various short time steps by using FDR method and the performance of heat flow calculation through building construction based on these CTF coefficients. The results of several typical wall constructions and a roof construction are presented and analyzed in detail.

2. Calculation of short time step CTF coefficients using FDR method

The principle of frequency-domain regression (FDR) method is briefed as follows for readers to follow up easily. FDR method is to develop a simplified thermal model of a building construction for heat flow calculation based on the equivalent frequency characteristics of this simplified model and the coincident theoretical model [12,16]. The simplified model is a simple polynomial s-transfer function which is very easy to carry out the heat flow calculation through the building construction.

The transmission equation relating temperatures to heat flows on both sides (inside and outside) of a multilayer construction in s-domain is given by Equation (1). The external, cross and internal transfer functions (i.e., $G_X(s)$, $G_Y(s)$ and $G_Z(s)$) of the multilayer construction can be expressed as Equations (2–4). $A(s)$, $B(s)$ and $D(s)$ are the elements of the transmission matrix of the construction which are complex hyperbolic functions. Calculation of the thermal response factors and CTF coefficients requires finding the poles of the complex hyperbolic function of $B(s)$. This is a computationally lengthy and tedious process [8]. In FDR method, these three exact complex hyperbolic transfer functions $G_X(s)$, $G_Y(s)$ and $G_Z(s)$ are approximately represented using $G'_X(s)$, $G'_Y(s)$ and $G'_Z(s)$ respectively, which are the form of a ratio of two polynomials as Equations (5–7). This approximation can make the root-finding process very easy since it is very easy to find the poles of a polynomial transfer function.

$$\begin{bmatrix} q_{\mathrm{out}}(s) \\ q_{\mathrm{in}}(s) \end{bmatrix} = \begin{bmatrix} -G_X(s) & G_Y(s) \\ -G_Y(s) & G_Z(s) \end{bmatrix} \begin{bmatrix} T_{\mathrm{out}}(s) \\ T_{\mathrm{in}}(s) \end{bmatrix} \quad (1)$$

$$G_X(s) = A(s)/B(s) \quad (2)$$

$$G_Y(s) = 1/B(s) \quad (3)$$

$$G_Z(s) = D(s)/B(s) \tag{4}$$

$$G'_X(s) = B_X(s)/A_X(s) \tag{5}$$

$$G'_Y(s) = B_Y(s)/A_Y(s) \tag{6}$$

$$G'_Z(s) = B_Z(s)/A_Z(s) \tag{7}$$

Where, q is heat flow, T is temperature, $B_X(s)$, $A_X(s)$, $B_Y(s)$, $A_Y(s)$, $B_Z(s)$, $A_Z(s)$ are polynomials.

When these approximate transfer functions are regressed independently in terms of their frequency responses equivalent to their coincident theoretical frequency responses, it is obvious that $A_X(s) \neq A_Y(s) \neq A_Z(s)$, which will result in three different sets of d_k in CTF coefficients. With conventional methods, such as direct root-finding method and state-space method [8,11], d_k derived from these three exact transfer functions $G_X(s)$, $G_Y(s)$ and $G_Z(s)$ are identical. To conform to the common understanding of a unique set of d_k in current simulation packages for heat flow calculation and energy analysis, a constraint is enforced in the regression process [16]. The regressed denominator $A_Y(s)$ is used as the benchmark, and let the dominators $A_X(s)$ and $A_Z(s)$ be equal to $A_Y(s)$, and then the coefficients of the numerators $B_X(s)$ and $B_Z(s)$ are identified subsequently.

With these identified polynomial s-transfer functions, the response factors and CTF coefficients can be derived easily. The response factors for heat transfer calculation are a series of discretized responses of the transfer functions of a construction to a unit triangular temperature pulse which is the combination of a ramp at time $t = -\Delta$ with a slope of $\eta = 1/\Delta\ \mathrm{Kh}^{-1}$, a ramp at $t = 0$ with a slope of $\eta = 2/\Delta\ \mathrm{Kh}^{-1}$, and a ramp $t = -\Delta$ with a slope of $\eta = 1/\Delta\ \mathrm{Kh}^{-1}$. It is noted that the time step Δ can be any time such as 1800 s, 900 s, 60 s, 1 s etc. as needed although 3600 s is usually used. The cross thermal response factors $Y(k)$ are obtained by applying the inverse Laplace transform to $G'_Y(s)/s^2$. The value of the factor $Y(0)$ at time $t = 0$ is calculated as Equation (8), and the value of the factor $Y(k)$ at time $t = k\Delta$ ($k = 1,2,3\ldots$) is calculated as Equation (9). The CTF coefficients in Equation (10) can be derived by directly performing z-transform on Equations (8) and (9) with the sampling time of Δ. The CTF coefficients associated with the internal transfer function can also be derived as Equation (11) similarly. The details of these derivations were presented in the Refs. [12,16]. With these CTF coefficients, the heat flow through a building construction at the inside surface is calculated as Equation (12).

$$Y(0) = U + \sum_{j=1}^{m} \sigma_j \left(1 - e^{s_j \Delta}\right) \Big/ \Delta \tag{8}$$

$$Y(k) = -\sum_{j=1}^{m} \sigma_j \left(1 - e^{s_j \Delta}\right)^2 e^{(k-1)s_j \Delta} / \Delta \quad (k = 1, 2, 3, \cdots) \tag{9}$$

$$G'_Y(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_r z^{-r}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_m z^{-m}} \tag{10}$$

$$G'_Z(z) = \frac{c_0 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_r z^{-r}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_m z^{-m}} \tag{11}$$

$$Q_i = \sum_{k=0}^{r} b_k T_{\mathrm{out},i-k} - \sum_{k=1}^{m} d_k Q_{i-k} - \sum_{k=0}^{r} c_k T_{\mathrm{in},i-k} \tag{12}$$

Where, U is the thermal transmittance of the construction, Δ is the time step which could be customized as needed, s_j are the roots of the denominator $A_Y(s)$, σ_j are the residues of $G'_Y(s)/s_j^2$ for the j-th root, b_k, c_k and d_k are CTF coefficients, Q is heat flow flux, T_{out} and T_{in} are outdoor air temperature (or solar air temperature) and indoor air temperature respectively.

Table 1
Details of the physical properties of these constructions of concern.

Description	Thickness and thermal properties				
	L(mm)	λ(W m^{-1} K^{-1})	ρ(kg m^{-3})	C_p(J kg^{-1} K^{-1})	R(m^2 K W^{-1})
Wall Group 2:					
Outside surface film					0.060
Stucco	25	0.692	1858	840	0.036
Insulation	125	0.043	91	840	2.907
Plaster or gypsum	20	0.727	1602	840	0.028
Inside surface film					0.120
Brick/cavity wall:					
Outside surface film					0.060
Brickwork	105	0.840	1700	800	0.125
Cavity					0.180
Heavyweight concrete	100	1.630	2300	1000	0.061
Inside surface film					0.120
Wall Group 41:					
Outside surface film					0.060
Face brick	100	1.333	2002	920	0.075
Insulation	125	0.043	91	840	2.907
High density concrete	300	1.731	2243	840	0.173
Plaster or gypsum	20	0.727	1602	840	0.028
Inside surface film					0.120
Roof construction:					
Outside surface film					0.060
Gravel surface	12	1.436	881	1.67	0.009
Built-up roofing	10	0.190	1121	1.67	0.050
Insulation	51	0.043	32	0.84	1.173
Mental deck	2	44.998	7689	0.42	0.000
Gypsum slab	100	0.173	641	0.84	0.587
Inside surface film					0.120

Table 2
CTF coefficients of Wall Group 2 at various time steps.

	k						Calculated U value
	0	1	2	3	4	5	
Time step $\Delta = 3600$ s							
b_k	9.356678E-04	3.165971E-02	5.460031E-02	1.193587E-02	2.768229E-04	3.041151E-07	0.317479
c_k	4.950058E+00	−7.483352E+00	2.971137E+00	−3.401356E-01	1.706114E-03	−5.092724E-06	0.317479
d_k	1.000000E+00	−9.355651E-01	2.738394E-01	−2.526493E-02	1.095122E-04	−3.532504E-08	
$\Delta = 1800$ s							
b_k	4.727864E-06	1.853722E-03	1.158368E-02	1.000414E-02	1.438610E-03	2.348765E-05	0.317479
c_k	6.080903E+00	−1.275364E+01	8.788538E+00	−2.234344E+00	1.460655E-01	−2.616704E-03	0.317479
d_k	1.000000E+00	−1.725393E+00	1.020708E+00	−2.306784E-01	1.400800E-02	−1.879496E-04	
$\Delta = 1200$ s							
b_k	−1.043896E-06	1.475665E-04	2.193781E-03	4.305893E-03	1.583787E-03	7.737372E-05	0.317479
c_k	6.560917E+00	−1.643947E+01	1.457871E+01	−5.422654E+00	7.648756E-01	−3.407112E-02	0.317479
d_k	1.000000E+00	−2.230848E+00	1.795907E+00	−6.175098E-01	8.189854E-02	−3.281161E-03	
$\Delta = 900$ s (*)							
b_k	5.730057E-06	−1.690524E-05	5.004844E-04	1.502294E-03	1.226144E-03	1.268203E-04	0.317477
c_k	6.828839E+00	−1.925408E+01	1.999423E+01	−9.300989E+00	1.864825E+00	−1.294843E-01	0.317110
d_k	1.000000E+00	−2.600472E+00	2.518532E+00	−1.104257E+00	2.104413E-01	−1.370947E-02	
$\Delta = 900$ s							
b_k	5.7300570820000000E-06	−1.6905242018000000E-05	5.0048444109100000E-04	1.5022937450190000E-03	1.2261438714340000E-03	1.2682033540700000E-04	0.317479
c_k	6.8288387800218800E+00	−1.9254079342299900E+01	1.9994233291381200E+01	−9.3009887574178100E+00	1.8648249028043500E+00	−1.2948430728178600E-01	0.317479
d_k	1.0000000000000000E+00	−2.6004724372372300E+00	2.5185318808296500E+00	−1.1042565081460300E+00	2.1044129942532600E-01	−1.3709469504233000E-02	
$\Delta = 600$ s (*)							
b_k	1.445008E-05	−7.594148E-05	1.830175E-04	7.711732E-06	4.688070E-04	1.867713E-04	0.317536
c_k	7.120910E+00	−2.327280E+01	2.911951E+01	−1.727687E+01	4.812082E+00	−5.020521E-01	0.315547
d_k	1.000000E+00	−3.110953E+00	3.723589E+00	−2.123080E+00	5.701970E-01	−5.728142E-02	
$\Delta = 600$ s							
b_k	1.4450082054000000E-05	−7.5941477836000000E-05	1.8301749466900000E-04	7.7117321880000000E-06	4.6880700494100000E-04	1.8677134600600000E-04	0.317479
c_k	7.1209097223187200E+00	−2.3272797584786600E+01	2.9119509811000100E+01	−1.7276866686590200E+01	4.8120816299952600E+00	−5.0205207575519200E-01	0.317479
d_k	1.0000000000000000E+00	−3.1109526346743800E+00	3.7235891587642600E+00	−2.1230800818058900E+00	5.7019700170500300E-01	−5.7281418815035000E-02	
$\Delta = 300$ s (*)							
b_k	−1.127836E-05	9.493795E-05	−3.165406E-04	5.426042E-04	−4.961109E-04	2.324727E-04	0.318707
c_k	7.444271E+00	−2.934634E+01	4.568333E+01	−3.506161E+01	1.325239E+01	−1.972001E+00	0.276625
d_k	1.000000E+00	−3.855263E+00	5.876049E+00	−4.420216E+00	1.638910E+00	−2.393354E−01	
$\Delta = 300$ s							
b_k	−1.1278364687000000E-05	9.4937953521000000E-05	−3.1654059783000000E-04	5.4260417645300000E-04	−4.9611089211500000E-04	2.3247267151200000E-04	0.317479
c_k	7.4442706621486800E+00	−2.9346341782929500E+01	4.5683332991161200E+01	−3.5061606324666200E+01	1.3252391262651400E+01	−1.9720007234188000E+00	0.317479
d_k	1.0000000000000000E+00	−3.8552628196602600E+00	5.8760493869902200E+00	−4.4202157130679500E+00	1.6389096736769900E+00	−2.3933536891783200E−01	
$\Delta = 60$ s (*)							
b_k	−1.426411E-04	7.742094E-04	−1.686415E-03	1.843344E-03	−1.011445E-03	2.229736E-04	−0.129500
c_k	7.733101E+00	−3.670617E+01	6.965236E+01	−6.604659E+01	3.129529E+01	−5.927998E+00	35.000000
d_k	1.000000E+00	−4.727177E+00	8.933721E+00	−8.437176E+00	3.981911E+00	−7.512792E−01	
$\Delta = 60$ s							
b_k	−1.4264112060400000E-04	7.7420942176900000E-04	−1.6864153014470000E-03	1.8433437728770000E-03	−1.0114449805710000E-03	2.2297363380500000E-04	0.317478
c_k	7.7331009302956100E+00	−3.6706166763563900E+01	6.9652364928075400E+01	−6.6046589748025800E+01	3.1295288676544400E+01	−5.9279979978992200E+00	0.317486
d_k	1.0000000000000000E+00	−4.7271766266727000E+00	8.9337205241252400E+00	−8.4371759352852300E+00	3.9819113379439200E+00	−7.5127922002427600E−01	

Table 3
Hourly heat flow through Wall Group 2 using CTF coefficients at various time steps.

i	$T_{sol\text{-}air}$	T_{in}	Q (W/m^2)										
			$\Delta = 3600$ s	$\Delta = 1800$ s	$\Delta = 1200$ s	$\Delta = 900$ s (*)	$\Delta = 900$ s	$\Delta = 600$ s (*)	$\Delta = 600$ s	$\Delta = 300$ s (*)	$\Delta = 300$ s	$\Delta = 60$ s (*)	$\Delta = 60$ s
0	25.0	25.0	−7.56255	−7.48256	−7.46918	−7.45575	−7.46404	−7.41496	−7.45990	−6.49626	−7.45825	*N/A*	−7.45775
1	24.4	25.6	−5.52887	−5.41157	−5.39107	−5.37487	−5.38343	−5.33210	−5.37790	−4.40459	−5.37484		−5.37401
2	24.4	25.9	−3.65168	−3.54511	−3.52654	−3.51070	−3.51955	−3.46755	−3.51460	−2.51840	−3.51173		−3.51099
3	23.8	26.0	−2.14086	−2.07118	−2.05989	−2.04638	−2.05549	−2.00402	−2.05240	−1.02925	−2.05065		−2.05028
4	23.3	26.1	−1.40545	−1.38479	−1.38261	−1.37212	−1.38141	−1.33108	−1.38060	−0.33380	−1.38024		−1.38031
5	23.3	26.1	−1.20213	−1.16654	−1.15982	−1.14745	−1.15686	−1.10436	−1.15480	−0.08848	−1.15354		−1.15335
6	23.8	26.0	−0.45520	−0.42224	−0.41819	−0.40690	−0.41638	−0.36422	−0.41530	0.66303	−0.41453		−0.41453
7	25.5	25.8	0.32758	0.42862	0.44971	0.46742	0.45790	0.51512	0.46370	1.55262	0.46737		0.46831
8	27.2	25.1	3.22542	3.36085	3.38335	3.40107	3.39155	3.44875	3.39710	4.48947	3.40069		3.40157
9	29.4	24.3	5.17753	5.20875	5.21290	5.22417	5.21474	5.26732	5.21590	6.30406	5.21669		5.21671
10	31.6	23.6	5.90526	5.85836	5.84707	5.85266	5.84338	5.89152	5.84060	6.91735	5.83896		5.83821
11	33.8	23.2	5.21810	5.09767	5.07321	5.07400	5.06493	5.10898	5.05900	6.11711	5.05528		5.05391
12	36.1	23.0	4.44913	4.44842	4.45092	4.46141	4.45255	4.50285	4.45400	5.49551	4.45457		4.45463
13	43.3	22.6	5.59768	5.52271	5.50249	5.50396	5.49527	5.53798	5.49010	6.50727	5.48651		5.48522
14	49.4	22.7	4.22247	4.12565	4.11118	4.11547	4.10694	4.15134	4.10430	5.10595	4.10233		4.10159
15	53.8	22.6	5.58488	5.53895	5.52526	5.52887	5.52044	5.56349	5.51730	6.50436	5.51452		5.51357
16	55.0	22.8	5.65408	5.62108	5.61764	5.62553	5.61718	5.66325	5.61780	6.59818	5.61681		5.61657
17	52.7	22.8	7.49023	7.52828	7.53185	7.54156	7.53324	7.58029	7.53550	8.51261	7.53454		7.53446
18	45.5	22.9	8.14045	8.20708	8.22148	8.23549	8.22718	8.27761	8.23330	9.21354	8.23382		8.23441
19	30.5	22.6	10.19708	10.44555	10.49205	10.51722	10.50892	10.56748	10.52350	11.51231	10.52821		10.53031
20	29.4	22.0	11.52094	11.57491	11.57966	11.58952	11.58123	11.62884	11.58500	12.57131	11.58257		11.58248
21	28.3	22.0	7.70644	7.55262	7.52351	7.52197	7.51375	7.55340	7.50920	8.49066	7.50257		7.50102
22	27.2	22.3	3.74560	3.54046	3.49792	3.49120	3.48303	3.51842	3.47390	4.44520	3.46565		3.46334
23	26.1	23.5	−2.89408	−3.36879	−3.46412	−3.48958	−3.49774	−3.47637	−3.52100	−2.56841	−3.53677		−3.54169
	$Q_{Total,\Delta}$ (J)		249,559	249,627	249,550	250,317	249,559	253,717	249,620	337,546	249,559	Divergence	249,541
	Error (%)			0.03	0.00	0.30	0.00	1.67	0.02	35.22	0.00		−0.01

Remark: *indicates the heat flow was calculated using CTF with the precision to six places of decimals with E-notation.

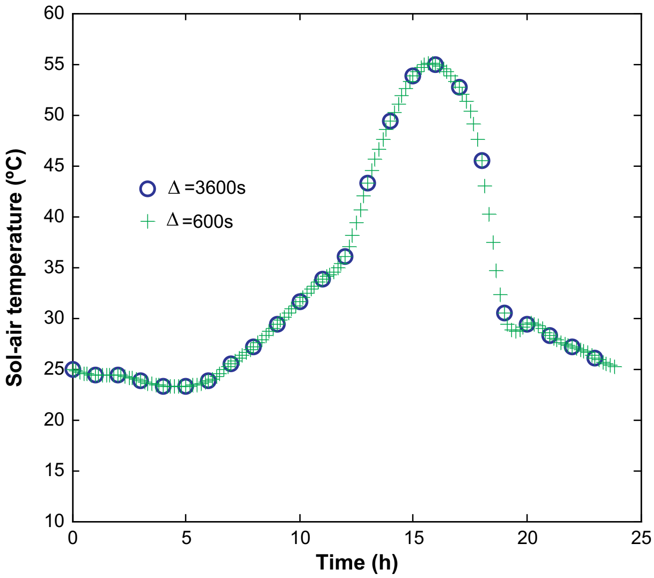

Fig. 1. Profiles of hourly sol-air temperature and sol-air air temperature at the time step of 600 s using cubic spline interpolation method.

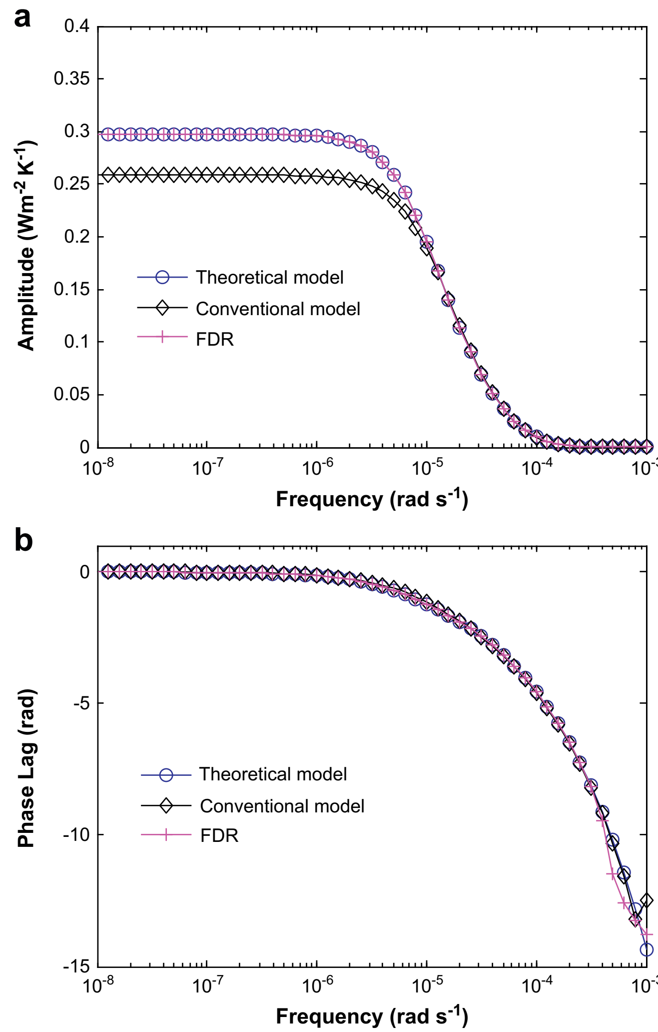

Fig. 3. Frequency response of the cross heat conduction of Wall Group 41 (a) Amplitude; (b) Phase lag.

3. Case studies and analysis

Many cases were carried out to investigate the performance of short time step heat flow calculation through building constructions using FDR method. Only the results of several representative building constructions representing light-weighted constructions, medium-weighted walls and heavy-weighted walls are presented to demonstrate the reliability and accuracy of the short time step heat flow calculation using FDR method. These constructions are three typical exterior walls and one roof. The first wall is Wall Group 2 selected from *ASHRAE Handbook of fundamentals* [23]. This wall, which is mainly the insulation layer, is a typical-light weighted wall with the density 90 kg/m^2. The second wall is a brick/cavity wall, which is often used to validate various methods for heat transfer calculation. It is a medium-weighted construction with the density 409 kg/m^2. The third wall is Wall group 41 in *ASHRAE Handbook of fundamentals* [23]. This wall, mainly composed of insulation and high density concrete, is a typical heavy-weighted wall with the density 917 kg/m2. The roof construction is also selected from *ASHRAE Handbook of fundamentals* [23], where it was used as an example for the cooling load calculation of a small office building. It is a light-weighted construction with the density of 146 kg/m^2. The detailed properties of these four constructions are list in Table 1.

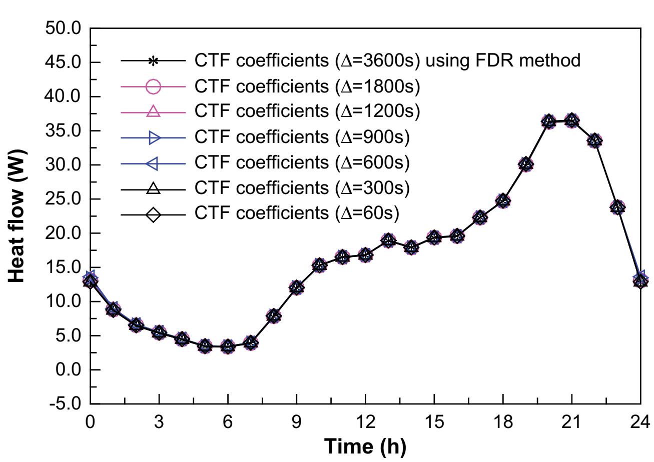

Fig. 2. Hourly heat flows through the brick/cavity wall using CTF coefficients at various time steps.

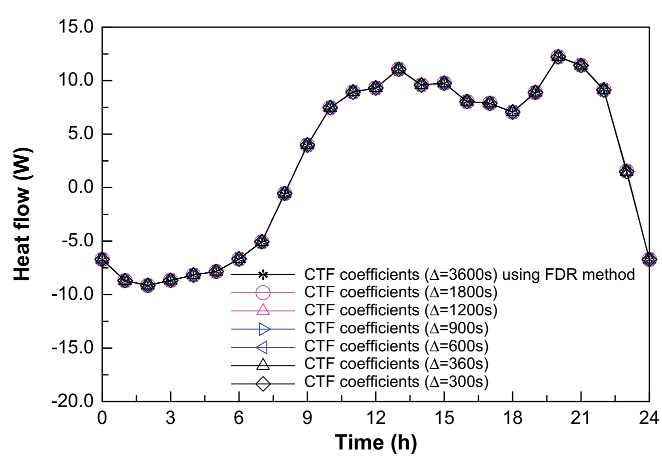

Fig. 4. Hourly heat flows through Wall Group 41 using CTF coefficients at various time steps.

The equivalent polynomial s-transfer functions of Wall Group 2 were found easily by using FDR method. The polynomial s-transfer functions of cross and internal heat conductions as (13) and (14) were used for generating CTF coefficients at various time steps as presented in Table 2. The transient heat flows through this construction at these time steps were calculated using these CTF coefficients with varying solar air temperature and varying indoor air temperature simultaneously. Hourly sol-air temperature and varying indoor air temperature are presented in Table 3. For the heat flow calculation at short time steps rather than one hour, these temperatures are needed to be interpolated. In this study, cubic spline interpolation method was used. Fig. 1 presents the comparison of the hourly sol-air temperature and the sol-air temperature at the time step of 600 s. The comparison shows the interpolated temperatures could follow the trend of these hourly temperatures very well.

$$G'_Y(s) = \frac{-2.09287\times10^{-4}s^5 - 1.54629\times10^{-6}s^4 - 6.78347\times10^{-9}s^3 - 2.01040\times10^{-11}s^2 - 3.87181\times10^{-14}s - 3.76813\times10^{-17}}{s^5 - 4.76630\times10^{-3}s^4 + 7.56436\times10^{-6}s^3 - 4.78410\times10^{-9}s^2 + 1.27431\times10^{-12}s - 1.18689\times10^{-16}} \quad (13)$$

$$G'_Z(s) = \frac{7.81032s^5 - 3.46034\times10^{-2}s^4 + 4.86272\times10^{-5}s^3 - 2.36324\times10^{-9}s^2 + 3.68964\times10^{-12}s - 3.76488\times10^{-17}}{s^5 - 4.76630\times10^{-3}s^4 + 7.56436\times10^{-6}s^3 - 4.78410\times10^{-9}s^2 + 1.27431\times10^{-12}s - 1.18689\times10^{-16}} \quad (14)$$

For the evaluation of the performance of heat flow calculation at various time steps, two criterions are used. The first is to compare the calculated heat flows at integer hour. The other is to compare the daily total heat flows calculated at various time steps as Equation (15) with the daily total heat flow calculated at the time step of 3600 s. The error is defined as Equation (16). It is noted that the CTF series at the time step of 3600 s calculated using conventional methods or FDR method for heat flow calculation are relatively reliable and accurate. Therefore, heat flow calculated at the time step of 3600 s is used as a benchmark for evaluating the heat flow calculation at other time steps. The heat flows at integer hour calculated at various time steps are presented in Table 3. The results show these heat flows at integer hour are very similar. The errors are also very small taking the daily total heat flow calculated at the time step of 3600 s as the benchmark. The heat flow calculated using the CTF coefficients at the time step of 60 s are stable and accurate. The time step of 60 s is sufficient for the purpose of heat flow calculation through building constructions for control and/or optimization simulation of air-conditioning systems.

$$Q_{\text{Total},\Delta} = \Delta\sum_{i=1}^{n} Q_i \quad (15)$$

$$\text{Error} = \frac{Q_{\text{Total},\Delta} - Q_{\text{Total},3600}}{Q_{\text{Total},3600}} \times 100\% \quad (16)$$

It is noted that CTF coefficients list in Refs. [8,13,23] usually keep precision to five or six places of decimals. Such precisions of CTF coefficients for heat flow calculation at short time steps may result in big deviations from the real values due to the leading zeros after the decimal and the truncated error. In this study, the precision to six places of decimals with E-notation was used. The results show that slight deviation occurs with the error of 1.67% when the time step is 600 s, and significant deviation occurs with the error of 35.22% as shown in Table 3 when the time step is 300 s. When double precision of these CTF coefficients for calculating the heat flow was used, the error of the calculated daily heat flow is very small at the time step of 600 s and 300 s. For the time step of 60 s, the precision of CTF coefficients to six places of decimals with E-notation resulted in unreasonable U values as shown in Table 2, and the heat flow calculation using these CTF series was diverging. However, double precision of these CTF coefficients for calculating the U values and the heat flow resulted in enough accuracy as shown in Table 2 and Table 3 respectively.

For the brick/cavity wall, the polynomial s-transfer functions for cross and internal heat transfers can also be identified as shown in the Ref. [16]. Based on these two transfer functions, the CTF coefficients at various time steps were also deduced. The detailed coefficients at various time steps are not presented for avoiding overmuch occupation of space. The sol-air temperature and the varying indoor air temperature list in Table 3 were also used for the heat flow calculation at various time steps. The heat flows at integer hour calculated using these CTF coefficients are illustrated in Fig. 2. The results show the heat flows at integer hour overlapped. It is noted that the calculated daily heat flows have significant errors of −3.10% and 20.57% at the time step of 600 s and 300 s respectively when the precision to six places of decimals with E-notation was used. Therefore, double precision is needed for ensuring the reliability and accuracy of heat flow calculation. It is worth to point out that the heat flow calculation is still stable and also very accurate even if the time step is as short as 60 s.

The equivalent polynomial s-transfer functions of cross and internal heat transfers of Wall Group 41 were identified using FDR method, and presented as Equations (17) and (18). The CTF coefficients at various time steps were also calculated. The CTF coefficients of this construction list in the Ref. [23] were used to calculate the frequency response of the cross heat conduction. Fig. 3 presents the theoretical frequency response of the cross heat conduction (i.e., theoretical model), the frequency response calculated using the list b_k and d_k (i.e., conventional model), and the frequency response calculated based on the polynomial s-transfer function (i.e., FDR). The results show that the equivalent polynomial s-transfer function can represent the dynamic thermal characteristic of the cross heat conduction much better than the list CTF coefficients in the Ref. [23]. On the other hand, the calculated U values based on the list coefficients are also less than the real value. These CTF coefficients and U values calculated at various time steps using FDR method are not presented for conciseness.

$$G'_Y(s) = \frac{-1.59637 \times 10^{-4}s^5 - 1.79821 \times 10^{-7}s^4 - 1.14646 \times 10^{-10}s^3 - 4.87066 \times 10^{-14}s^2 - 1.33153 \times 10^{-17}s - 1.82825 \times 10^{-21}}{s^5 - 7.78558 \times 10^{-4}s^4 + 2.13109 \times 10^{-7}s^3 - 2.28681 \times 10^{-9}s^2 + 8.75109 \times 10^{-16}s - 6.14660 \times 10^{-21}} \quad (17)$$

$$G'_Z(s) = \frac{7.01038s^5 - 4.88251 \times 10^{-2}s^4 + 1.20886 \times 10^{-6}s^3 - 1.12364 \times 10^{-10}s^2 + 3.43726 \times 10^{-15}s - 1.82743 \times 10^{-21}}{s^5 - 7.78558 \times 10^{-4}s^4 + 2.13109 \times 10^{-7}s^3 - 2.28681 \times 10^{-9}s^2 + 8.75109 \times 10^{-16}s - 6.14660 - \times 10^{-21}} \quad (18)$$

The heat flows through this heavy construction were calculated using the CTF coefficients at various time steps based on FDR method. Fig. 4 presents the heat flows calculated using CTF coefficients at various time steps. The results show these heat flows agreed very well. It is noted that the CTF coefficients need to be of high precision for ensuring the stability and accuracy of heat flow calculation when the time step is less than 3600 s. When the precision to six places of decimals with E-notation was used for the time step of 1800 s, the calculated heat flow error is 128.85%. It is also noted that these CTF coefficients is unreasonable and the heat flow calculation was diverging at the time step of 60 s even if double precision was used.

$$G_Y(s) = \frac{-2.68605 \times 10^{-4}s^5 - 1.49550 \times 10^{-6}s^4 - 5.03428 \times 10^{-9}s^3 - 1.13203 \times 10^{-11}s^2 - 1.64142 \times 10^{-14}s - 1.19556 \times 10^{-17}}{s^5 - 3.74130 \times 10^{-4}s^4 + 4.88922 \times 10^{-7}s^3 - 2.57361 \times 10^{-9}s^2 + 5.20940 \times 10^{-13}s - 2.39844 \times 10^{-17}} \quad (19)$$

$$G_Z(s) = \frac{6.54739s^5 - 1.94783 \times 10^{-2}s^4 + 2.08638 \times 10^{-5}s^3 - 8.39073 \times 10^{-9}s^2 + 1.16032 \times 10^{-12}s - 1.17386 \times 10^{-17}}{s^5 - 3.74130 \times 10^{-4}s^4 + 4.88922 \times 10^{-7}s^3 - 2.57361 \times 10^{-9}s^2 + 5.20940 \times 10^{-13}s - 2.39844 \times 10^{-17}} \quad (20)$$

Roof construction is chosen for analysis due to that a roof usually experiences significantly higher sol-air temperature (i.e., more high frequency disturbance) than a wall does. The light roof construction list in Table 1 is a 115 mm flat roof of 50 mm gypsum slab on metal roof deck, 50 mm rigid roof insulation, surfaced with two layers of mopped felt vapor-seal built-up roofing having dark-colored gravel surface, and with no false ceiling below underside of roof deck. *ASHRAE Handbook of fundamentals* [23] used it for a small office building as an example for cooling load calculation (i.e., Example 6 in Chapter 28.33). This Ref. provides the CTF coefficients (i.e., b_k, Σc, and d_k), and the sol-air temperature on this construction, and the hourly total heat gains. These CTF coefficients were duplicated in Table 4 to evaluate the frequency characteristics of the cross heat conduction. The hourly heat gain per square meter was calculated by dividing the list hourly total heat gain by the coincident area.

The equivalent polynomial *s*-transfer functions of cross and internal heat conductions are shown as Equations (19) and (20). To evaluate the validity of the published CTF coefficients in the Ref. [23] and the regressed polynomials, numerical comparisons were made within the frequency range of normal concern (10^{-8}–10^{-3} rads^{-1}) among the frequency response of equivalent polynomial *s*-transfer functions (i.e., FDR), the frequency response of CTF models in the above Ref. (i.e., conventional model) and their theoretical frequency responses. These frequency responses of the cross heat conduction are presented in Fig. 5 in terms of amplitude and phase lag. The results show that the frequency responses of the equivalent polynomial *s*-transfer function agreed with the theoretical frequency responses almost the same as those of the CTF models in the Ref. [23].

With these polynomials, the CTF coefficients were calculated at various time steps. Only the CTF coefficients with the precision to six places of decimals with E-notation at the time step of 3600 s are presented in Table 4 by comparing with the published CTF coefficients in Ref. [23]. The CTF coefficients at other time steps and different precisions are not presented for avoiding undue occupation of space. The sol-air temperature on this construction [23] and the constant indoor air temperature of 24 °C were used for heat flow calculation by using the calculated CTF coefficients at various time steps. The results show that the heat flows at integer hour are almost identical as presented in Fig. 6. The errors of the daily total heat flows are also very small. The heat flows from the Ref. [23] are also presented. The heat flows calculated at the time step of 3600 s by using FDR method agreed basically with the heat flows from the Ref. [23] with the daily total heat gain error of 2.36%. 2.36% is not a great value. However, one of the main advantages of FDR method is that it can produce accurate and reliable CTF coefficients at more short time step other than the steps mentioned in the Ref. [3].

As mentioned before, the precision of CTF coefficients may have impacts on the heat flow calculation when the time step becomes shorter. For this case, double precision needs for the CTF coefficients at the steps of 300 s and 60 s for ensuring stable and accurate heat flow calculation. It is also noted that the errors of the daily heat flow are 2.22% and 63.26% respectively for the time steps of 600 s and 300 s by using CTF coefficients having the precision to six places of decimals with E-notation. However, the errors are 0.00% and 0.32% respectively when double precision was used. When the time step is as short as 60 s, the precision to six places of decimals with E-notation resulted in diverging heat flow calculation while

Table 4
CTF coefficients of the roof construction at various time steps.

	k						Calculated *U* value
	0	1	2	3	4	5	
Fundalmental [23] (Real *U* value = 0.4985)							
b_k	0.00052	0.02372	0.04432	0.00929	n/a	n/a	0.508790
c_k $\sum = 0.07807$							0.510228
d_k	1.00000	−1.10395	0.26169	−0.00475	0.00002	0.00000	
FDR method							
Time step $\Delta = 3600$ s							
b_k	2.030039E-04	1.364513E-02	3.593391E-02	1.224514E-02	5.468139E-04	2.099465E-06	0.498476
c_k	3.554504E+00	−5.791090E+00	2.709953E+00	−4.213159E-01	1.074691E-02	−2.218900E-04	0.498476
d_k	1.000000E+00	−1.232016E+00	3.965366E-01	−3.927333E-02	2.889394E-04	−1.414600E-06	

double precision produced stable and accurate heat flow calculation. These results demonstrate that FDR method could not only compute accurate CTF coefficients at the conventional time step of 3600 s effectively but also the CTF coefficients at the other time steps while maintaining sufficient accuracy for heat flow calculation.

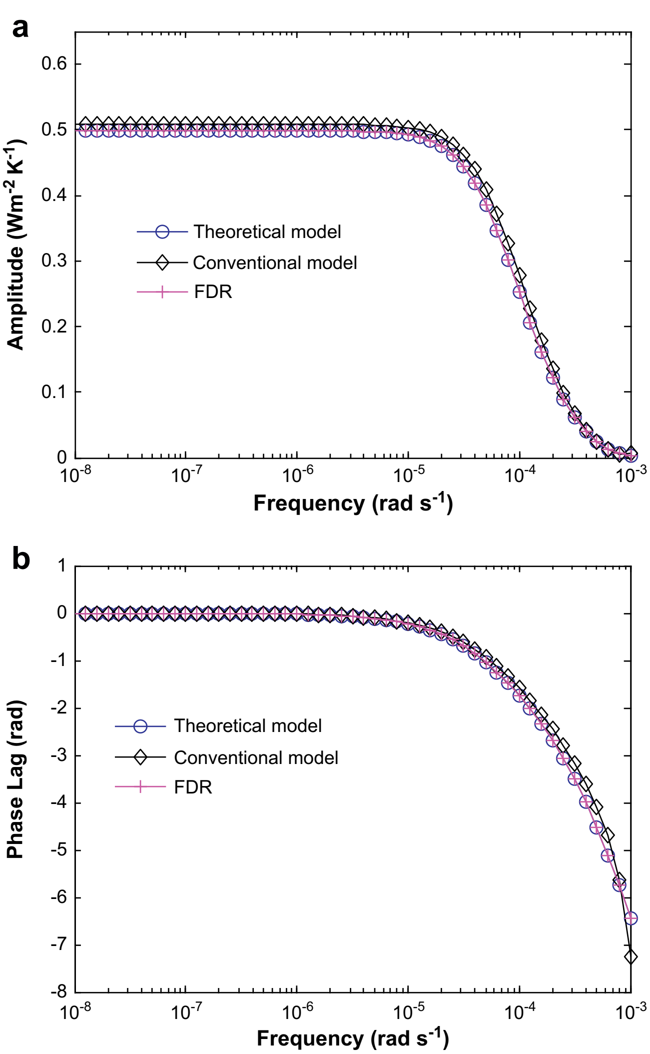

Fig. 5. Frequency response of the cross heat conduction of the roof construction (a) Amplitude; (b) Phase lag.

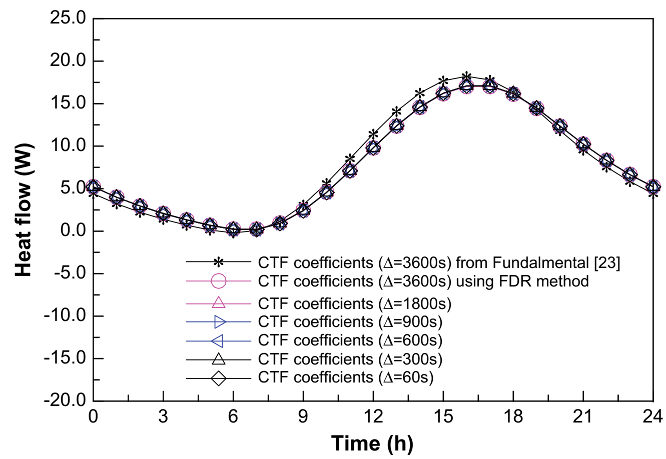

Fig. 6. Hourly heat flows through the roof construction using CTF coefficients with different methods.

4. Conclusion

This paper presents the performance of short time step heat flow calculation of representative building constructions based on FDR method. The results show that the regressed polynomial *s*-transfer functions may achieve stable CTF coefficients at various time steps even at the step of 60 s for light-weighted constructions and medium-weighted constructions, and these CTF coefficients can also produce heat flow flux through these building constructions with good accuracy. For heavy-weighted constructions, the CTF coefficients at the time step of 300 s are still stable, and can produce heat flow calculation of high accuracy.

The analysis on the frequency responses of heat conductions of some building constructions further show that FDR method can produce equivalent polynomial *s*-transfer functions well representing the real building construction. These polynomial *s*-transfer functions can result in stable CTF coefficients at various time steps for accurate heat flow calculation for practical applications such as control simulation of air-conditioning systems. The number of terms of CTF coefficients at various time steps computed using FDR method is usually 6, which is determined by the order of the regressed polynomial while it does not depend on time steps used. However, the number of terms in CTF series calculated using state-space method or root-finding method etc. grows as the time step gets shorter, and eventually the series becomes unstable [3]. Double precision should be used for CTF coefficients when the time step is short since the precision of CTF coefficients may have impacts on the accuracy of heat flow calculation. Cautions should also be taken when heavy constructions are of concern even if double precision is used.

Acknowledgement

The research work presented in this paper is financially supported by a grant (PolyU 5283/05E) from the Research Grants Council (RGC) of the Hong Kong SAR and the National Nature Sciences Foundation of China (50378033).

References

[1] D.G. Stephenson, G.P. Mitalas, Calculation of heat conduction transfer functions for multilayer slabs, ASHRAE Transaction 77 (2) (1971) 117–126.

[2] Lawrence Berkeley Laboratory, DOE-2 Engineering Manual Version 2.1C, Lawrence Berkeley Laboratory, Berkeley, CA, 1982.

[3] EnergyPlus Available from: http://www.eere.energy.gov/buildings/energyplus/ (2006).

[4] Blast, Building Loads Analysis and System Thermodynamics (BLAST), User's Manual, Version 3, University of Illinois, Urbana-Champain, Blast Support Iffice, IL, USA, 1986.

[5] G.E. Cossali, The heat storage capacity of a periodically heated slab under general boundary conditions, International Journal of Thermal Sciences 46 (9) (2007) 869–877.

[6] G.E. Cossali, Periodic heat conduction in a solid homogeneous finite cylinder, International Journal of Thermal Sciences 48 (4) (2009) 722–732.

[7] X.H. Xu, S.W. Wang, A simplified dynamic model for existing buildings using CTF and thermal network models, International Journal of Thermal Sciences 47 (9) (2008) 1249–1262.

[8] T. Kusuda, Thermal response factors for multilayer structures of various heat conduction system, ASHRAE Transactions 75 (1) (1969) 246–271.

[9] G.E. Myers, Long-time solutions to heat conduction transients with time-dependent inputs, Journal of Heat Transfer 102 (1980) 115–120.

[10] Y. Jiang, State-space method for the calculation of air-conditioning loads and the simulation of thermal behavior of the room, ASHRAE Transactions 88 (2) (1982) 122–138.

[11] J.E. Seem, Modeling of Heat Transfer in Buildings, Ph.D. Thesis. Madison: Department of Mechanical Engineering, University of Wisconsin, Madison, 1987.

[12] Y.M. Chen, S.W. Wang, Frequency-domain regression method for estimating CTF model of building multilayer constructions, Applied Mathematic Modeling 25 (7) (2001) 579–592.

[13] S.W. Wang, Y.M. Chen, Transient heat flow calculation for multilayer constructions using a frequency-domain regression method, Building and Environment 38 (1) (2003) 45–61.

[14] Y.M. Chen, S.W. Wang, A new procedure for calculating periodic response factors based on frequency domain regression method, International Journal of Thermal Sciences 44 (4) (2005) 382–392.

[15] Y.M. Chen, S.W. Wang, Z. Zuo, An approach to calculate transient heat flow through multilayer spherical structures, International Journal of Thermal Sciences 42 (8) (2003) 805–812.

[16] X.H. Xu, S.W. Wang, Y.M. Chen, An improvement to frequency-domain regression method for calculating conduction transfer functions of building walls, Applied Thermal Engineering 28 (2008) 661–667.

[17] X.Q. Li, Y.M. Chen, J.D. Spitler, et al., Applicability of calculation methods for conduction transfer function of building constructions, International Journal of Thermal Sciences 48 (7) (2009) 1441–1451.

[18] T.D. Taylor, C.O. Pedersen, D.E. Fisher et al., Impact of simultaneous simulation of buildings and mechanical systems in heat balance based energy analysis programs on system response and control, in: Conference Proceedings IBPSA Building Simulation '91, August 20-22, Nice, France, 1991.

[19] M. Zaheer-Uddin, G.R. Zheng, A dynamic model of multizone VAV system for control analysis, ASHRAE Transactions 100 (l) (1994) 219–229.

[20] J.M. House, A system approach to optimal control for HVAC and building systems, ASHRAE Transactions 101 (2) (1995) 647–660.

[21] S.W. Wang, Dynamic simulation of building VAV air-conditioning system and evaluation of EMCS on-line control strategies, Building and Environment 34 (6) (1999) 681–705.

[22] M. Mossolly, K. Ghali, N. Ghaddar, Optimal control strategy for a multi-zone air conditioning system using a genetic algorithm, Energy 34 (1) (2009) 58–66.

[23] Ashrae, ASHRAE Handbook-Fundamentals, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Atlanta, 1997.